Olimpiada de Matematică –etapa locală- Galaţi

19 februarie-2017

Clasa a VII-a

Barem de evaluare

- **Pentru orice soluţie corectă, chiar dacă este diferită de cea din barem, se acordă punctajul maxim corespunzător.**
- **Nu se acordă fracţiuni de punct, dar se pot acorda punctaje intermediare pentru rezolvări parţiale, în limitele punctajului indicat în barem.**

Nr. problemei	**Soluţie, rezolvare**	**Punctaj**
1.	a) $\frac{1}{1\cdot 2}=\frac{1}{1}-\frac{1}{2};\frac{1}{2\cdot 3}=\frac{1}{2}-\frac{1}{3};...;\frac{1}{2017\cdot 2018}=\frac{1}{2017}-\frac{1}{2018};$ $a=2\cdot\frac{2017\cdot 2018}{2}\cdot\left(\frac{1}{1}-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-...+\frac{1}{2017}-\frac{1}{2018}\right)=2017\cdot 2018\cdot\left(1-\frac{1}{2018}\right)=$ $2017\cdot 2018\cdot\frac{2017}{2018}=2017^2;$ $\sqrt{2017^2}=2017\in\mathbb{N}.$	1p 1p 1p
	$b)$ $b=\sqrt{2^0+2^1+2^2+...+2^{2017}}.$ $\left.\begin{array}{l}S=2^0+2^1+2^2+...+2^{2017}\\ 2\cdot S=2^1+2^2+...+2^{2017}+2^{2018}\end{array}\right\}\Rightarrow S=2^{2018}-1\Rightarrow b=\sqrt{2^{2018}-1};$	1p
	$u\left(2^{2018}\right)=4\Rightarrow u\left(2^{2018}-1\right)=3;$	1p
	$\left.\begin{array}{l}\text{Dar } u\left(k^2\right)\in\{0,1,4,5,6,9\}\\ u\left(2^{2018}-1\right)=3\end{array}\right\}\Rightarrow$ $\Rightarrow 2^{2018}-1$ nu este pătrat perfect	1p
	$\Rightarrow b=\sqrt{2^{2018}-1}\in\mathbb{R}-\mathbb{Q}.$	1p

<table>
<tr><td rowspan="2">2.</td><td>$a)\ \frac{x-5}{2017}+\frac{x-3}{2019}=\frac{x-2017}{5}+\frac{x-2019}{3}\ /-2\Rightarrow$
$\left(\frac{x-5}{2017}-1\right)+\left(\frac{x-3}{2019}-1\right)=\left(\frac{x-2017}{5}-1\right)+\left(\frac{x-2019}{3}-1\right)\Leftrightarrow$
$\frac{x-2022}{2017}+\frac{x-2022}{2019}=\frac{x-2022}{5}+\frac{x-2022}{3}\Leftrightarrow$
$(x-2022)\cdot\left(\frac{1}{2017}+\frac{1}{2019}\right)=(x-2022)\cdot\left(\frac{1}{5}+\frac{1}{3}\right)\Rightarrow$
$(x-2022)\cdot\left(\frac{1}{2017}+\frac{1}{2019}\right)-(x-2022)\cdot\left(\frac{1}{5}+\frac{1}{3}\right)=0\Rightarrow$
$(x-2022)\cdot\left(\frac{1}{2017}+\frac{1}{2019}-\frac{1}{5}-\frac{1}{3}\right)=0\Rightarrow x=2022;$
$\frac{1}{2017}+\frac{1}{2019}-\frac{1}{5}-\frac{1}{3}\neq 0$
Aşadar, $x=2022$ soluţie.</td><td>2p
1p
1p</td></tr>
<tr><td>$b)\ \underbrace{11111...111}_{2017\text{cifre de }1}-2017=\underbrace{11111...111}_{2012\text{cifre de }1}09094=$
$=3\cdot\underbrace{370370...370}_{670\text{ grupe}=370}369698;$
Notăm cu A=$\underbrace{370370...370}_{670\text{ grupe}=370}369698$.
Suma cifrelor numărului A este:
$(3+7)\cdot 670+3+6+9+6+9+8=6700+41=6741\vdots 9;$
$(3\cdot A)\vdots 27.$</td><td>1p
1p
1p</td></tr>
</table>

3.	Soluție: În $\triangle$ABC: $m(\measuredangle ABC)=90°$, $[BA]=[BC]$, $[AO]\equiv[OC]$ $\Rightarrow \begin{cases} BO \perp AC \\ m(\measuredangle ABO)=m(\measuredangle CBO)=m(\measuredangle CAB)=m(\measuredangle BCA)=45° \end{cases}$	**2p**
	Din $[AP$ bisectoarea $\measuredangle CAB$, $m(\measuredangle BAC)=45°$ $\Rightarrow m(\measuredangle PAB)=22°30'$	**1p**
	$m(\measuredangle PAB)=22°30'$, $m(\measuredangle ABC)=90°$ $\Rightarrow m(\measuredangle APB)=67°30'$ $m(\measuredangle APB)=67°30'$, $m(\measuredangle CBO)=45°$ $\Rightarrow m(\measuredangle PNB)=67°30'$	**1p**
	Fie OE $\parallel$ CB, E $\in (AP)$; $m(\measuredangle BNP)=m(\measuredangle ONE)=67°30'$ (unghiuri opuse la vârf), $m(\measuredangle BPN)=m(\measuredangle OEN)=67°30'$(unghiuri alterne interne) $\Rightarrow$ $m(\measuredangle ONE)=m(\measuredangle OEN)=67°30' \Rightarrow \triangle OEN$ este isoscel $\Rightarrow [OE]\equiv[ON]$ (1)	**1p**
	$[AO]\equiv[OC]$, OE $\parallel$ PC $\Rightarrow [OE]$ linie mijlocie în $\triangle$APC $\Rightarrow$ OE $\parallel$ BC, OE=$\frac{PC}{2}$; (2)	**1p**
	Din (1) şi (2) $\Rightarrow ON=\frac{PC}{2}$.	**1p**

4.	**Metoda 1.** Fie punctul M simetricul punctului B față de A $\Rightarrow$ M,A,B sunt coliniare și $[MA]\equiv[AB]$; Fie P $\in$ AD, AP=AD, A $\in$ (DP); $\triangle ANM \equiv \triangle ANP(L.U.L) \Rightarrow [MN]\equiv[NP]; m(\measuredangle ANM)=m(\measuredangle ANP)=30^\circ \Rightarrow m(\measuredangle MNP)=60^\circ$; $\left.\begin{array}{l}[MN]\equiv[NP]\\ m(\measuredangle MNP)=60^\circ\end{array}\right\} \Rightarrow \triangle MNP$ este echilateral $\Rightarrow [MN]\equiv[NP]\equiv[MP];(1)$ $\triangle AMP \equiv \triangle BAC(L.U.L) \Rightarrow [MP]\equiv[AC]; \quad (2)$ Din (1) și $(2) \Rightarrow [MN]\equiv[AC]$. **Metoda 2.** Fie punctul M simetricul punctului B față de A $\Rightarrow$ M,A,B sunt coliniare și $[MA]\equiv[AB]$; $\left.\begin{array}{l}MA \parallel DC\\ [MA]\equiv[DC]\end{array}\right\} \Rightarrow MACD -$ paralelogram $\Rightarrow AC \parallel DM$ și $[AC]\equiv[DM]$; *Fie* MN $\cap$ AD= $\{P\}$; $\left.\begin{array}{l}\text{Construim } AA' \perp DM\\ \triangle MAD \text{ -dreptunghic isoscel}\end{array}\right\} \Rightarrow AA' = \frac{MD}{2}$ $\left.\begin{array}{l}\text{Construim } PR \perp AC,\ R \in AC\\ PQ \perp MD, Q \in MD\\ MD \parallel AC\end{array}\right\} \Rightarrow Q,P,R$-coliniare $\left.\begin{array}{l}\triangle MAP\text{: } m(\measuredangle MAP)=90^\circ\\ m(\measuredangle PAM)=15^\circ\end{array}\right\} \Rightarrow \left.\begin{array}{l}m(\measuredangle MPA)=75^\circ\\ M-P-A\end{array}\right\} \Rightarrow \left.\begin{array}{l}m(\measuredangle APN)=105^\circ\\ m(\measuredangle PAN)=45^\circ\end{array}\right\} \Rightarrow m(\measuredangle ANP)=30^\circ$. $\left.\begin{array}{l}\triangle PRN\\ m(\measuredangle PRN)=90^\circ\\ m(\measuredangle RNP)=30^\circ\end{array}\right\} \Rightarrow PR=\frac{PN}{2} \quad (1)$ $\triangle MAD -$ dr.is. $\Rightarrow \left.\begin{array}{l}m(\measuredangle AMD)=45^\circ\\ m(\measuredangle PMA)=15^\circ\end{array}\right\} \Rightarrow m(\measuredangle PMD)=30^\circ$; $\left.\begin{array}{l}\triangle QMD\\ m(\measuredangle PQM)=90^\circ\\ m(\measuredangle PMQ)=30^\circ\end{array}\right\} \Rightarrow PQ=\frac{MP}{2} \quad (2)$ $\left.\begin{array}{l}\text{Din } (1) \text{ și } (2) \Rightarrow QR=PR+PQ=\frac{PN}{2}+\frac{MP}{2}=\frac{MN}{2}\\ AA'QR - \text{dreptunghi} \Rightarrow [AA']\equiv[QR];\ AA'=\frac{MD}{2}\end{array}\right\} \Rightarrow [MN]\equiv[MD]\equiv[AC]$.	**1p** **1p** **1p** **2p** **1p** **1p**