

**Olimpiada de Matematică –etapa locală- Galați**

**16 februarie-2020**

**Clasa a VII-a**

**Barem de evaluare**

- ◆ Pentru orice soluție corectă, chiar dacă este diferită de cea din barem, se acordă punctajul maxim corespunzător.
- ◆ Nu se acordă fracțiuni de punct, dar se pot acorda punctaje intermediare pentru rezolvări parțiale, în limitele punctajului indicat în barem.

Nr. problemei	Soluție, rezolvare	Punctaj
1.	<p>Notăm <math>d=(a,b) \Rightarrow</math> există <math>m, n \in \mathbb{N}^*</math> astfel încât <math>a=d \cdot m, b= d \cdot n, (m,n)=1</math>;</p> <p><math>3 \cdot [a,b]+5 \cdot (a,b)=123 \Rightarrow 3 \cdot d \cdot m \cdot n+5 \cdot d=123 \Rightarrow d \cdot (3 \cdot m \cdot n+5)=123 \Rightarrow d/123 \Rightarrow</math>  <math>d \in \{1, 3,41, 123\}</math>;</p> <p>Dar <math>5 \cdot d &lt; 123 \Rightarrow d \in \{1,3\}</math></p> <p>1. <math>d=1 \Rightarrow 3 \cdot m \cdot n+5=123 \Rightarrow 3m \cdot n=118 \Rightarrow 3/118 (F)</math></p> <p>2. <math>d=3 \Rightarrow 3 \cdot (3 \cdot m \cdot n+5)=123 \Rightarrow 3m \cdot n+5=41 \Rightarrow m \cdot n=12</math>;</p> <p><math>\left. \begin{matrix} mn=12 \\ (m,n)=1 \end{matrix} \right\} \Rightarrow \begin{cases} m=1 \\ n=12 \end{cases} \Rightarrow \begin{cases} a=3 \\ b=36 \end{cases}</math></p> <p><math>\left. \begin{matrix} m=3 \\ n=4 \end{matrix} \right\} \Rightarrow \begin{cases} a=9 \\ b=12 \end{cases}</math></p> <p><math>S = \{(3,36);(9,12)\}</math></p>	<p><b>1p</b></p> <p><b>1p</b></p> <p><b>1p</b></p> <p><b>2p</b></p> <p><b>1p</b></p> <p><b>1p</b></p>
2.	<p>a) <math>x=2019 \cdot a+b \cdot \sqrt{2019 \cdot 2020}+a \cdot \sqrt{2019 \cdot 2020}+2020 \cdot b \Leftrightarrow</math>  <math>x=2019 \cdot a+2020 \cdot b+(a+b) \cdot \sqrt{2019 \cdot 2020}</math>;</p> <p><math>\left. \begin{matrix} x \in \mathbb{Q} \\ a, b \in \mathbb{Z} \end{matrix} \right\} \Rightarrow a+b=0 \Rightarrow a=-b</math>;</p> <p><math>a^{2019}+b^{2019}=a^{2019}+(-a)^{2019}=a^{2019}-a^{2019}=0</math>;</p> <p><math>a^{2020}-b^{2020}=a^{2020}-(-a)^{2020}=a^{2020}-a^{2020}=0</math>.</p> <p>b) <math>\left. \begin{matrix} 2019 \cdot a+2020 \cdot b+(a+b) \cdot \sqrt{2019 \cdot 2020}=-1 \\ a, b \in \mathbb{Z} \end{matrix} \right\} \Rightarrow</math></p> <p><math>\left\{ \begin{matrix} a+b=0 \\ 2019 \cdot a+2020 \cdot b=-1 \end{matrix} \right. \Rightarrow \begin{cases} a=1 \\ b=-1 \end{cases}</math></p>	<p><b>2p</b></p> <p><b>1p</b></p> <p><b>1p</b></p> <p><b>1p</b></p> <p><b>2p</b></p>

3.	<p><math>ABCD</math> inscriptibil <math>\rightarrow \sphericalangle NDC \equiv \sphericalangle ABN</math> (1)</p> <p><math>\sphericalangle MDN + \sphericalangle MCN = 90^0 + 90^0 = 180^0 \rightarrow MDNC</math> inscriptibil  <math>\rightarrow \sphericalangle NDC \equiv \sphericalangle NMC</math> (2)</p> <p>Din (1) și (2) <math>\rightarrow \sphericalangle ABN \equiv \sphericalangle NMC</math></p> <p>Dar  <math>\sphericalangle NMC + \sphericalangle CNM = 90^0 \rightarrow \sphericalangle ABN + \sphericalangle BNM = 90^0 \rightarrow NM \perp AB</math> (3)</p> <p>Din <math>AP \parallel MC</math> și <math>MC \perp NB \rightarrow AP \perp NB</math> (4)</p> <p>Din (3), (4) și <math>MN \cap AP = \{Q\} \rightarrow Q =</math> ortocentrul triunghiului <math>\Delta ABN \rightarrow BQ \perp AN</math>.</p> <p>Cum <math>MD \perp AN \rightarrow BQ \parallel MD</math>.</p>	<p>2p</p> <p>1p</p> <p>1p</p> <p>2p</p> <p>1p</p>
4.	<p>Cazul 1. <math>m(\sphericalangle BAC) &lt; 90^0</math>  Fie <math>M</math> mijlocul lui <math>DC \Rightarrow DM=MC</math>;  <math>OB=OC \Rightarrow \Delta OBC</math> isoscel <math>\Rightarrow \sphericalangle OBD = \sphericalangle OCM</math>;  <math>DM=MC=BD=OD</math>  <math>\left. \begin{array}{l} \sphericalangle OBD = \sphericalangle OCM \\ OB=OC \end{array} \right\} \Rightarrow \Delta OBD = \Delta OCM \Rightarrow OD=OM=DM \Rightarrow</math>  <math>\Delta ODM</math> echilateral <math>\Rightarrow \sphericalangle OBD = BOD = OCM = MOC = 30^0</math>.  <math>OB = OA \Rightarrow \Delta OBA</math> isoscel  <math>\left. \begin{array}{l} \sphericalangle BOD=30^0 \text{ (unghi exterior } \Delta OAB) \end{array} \right\} \Rightarrow \sphericalangle OAB = OBA = 15^0 \Rightarrow \sphericalangle ABC = 45^0</math>  <math>OC = OA \Rightarrow \Delta OCA</math> isoscel  <math>\left. \begin{array}{l} \sphericalangle COD=90^0 \text{ (unghi exterior } \Delta OAC) \end{array} \right\} \Rightarrow \sphericalangle OAC = \sphericalangle OCA = 45^0 \Rightarrow</math>  <math>\sphericalangle ACB = 75^0</math> și <math>\sphericalangle CAB=60^0</math>.  Cazul 2. <math>m(\sphericalangle BAC) &gt; 90^0</math>  <math>\Delta ODM</math> echilateral <math>\Rightarrow \sphericalangle BDO = 120^0, \sphericalangle BOD=30^0 \Rightarrow \sphericalangle DAB = ABO = 75^0 \Rightarrow \sphericalangle ABC = 45^0</math>  <math>\Rightarrow \sphericalangle ACB = 15^0, \sphericalangle BAC=120^0</math>.</p>	<p>1p</p> <p>1p</p> <p>1p</p> <p>1p</p> <p>1p</p> <p>1p</p>

