**Olimpiada de Matematică –etapa locală- Galaţi**

**24 februarie 2019**

**Clasa a VIII-a**

**Barem de evaluare**

- ♦ **Pentru orice soluţie corectă, chiar dacă este diferită de cea din barem, se acordă punctajul maxim corespunzător.**
- ♦ **Nu se acordă fracţiuni de punct, dar se pot acorda punctaje intermediare pentru rezolvări parţiale, în limitele punctajului indicat în barem.**

| **Nr. problemei** | **Soluţie, rezolvare** | **Punctaj** |
|---|---|---|
| **1.** | a) $\sqrt{4n^2+n}<2n+1,(\forall)n\in\mathbb{N}$<br>Ridicând ambii membri la puterea a doua, se obţine:<br>$4n^2+n<(2n+1)^2 \Leftrightarrow 4n^2+n<4n^2+4n+1 \Leftrightarrow 0<3n+1$ (A), $(\forall)n\in\mathbb{N}$ | **2p** |
| | b)<br>Se demonstrează inegalitatea $2n\le\sqrt{4\cdot n^2+n},(\forall)n\in\mathbb{N}$. | **1p** |
| | Deci $2n\le\sqrt{n\cdot(4n+1)}<2n+1,\ (\forall)n\in\mathbb{N}\Rightarrow\left[\sqrt{n\cdot(4\cdot n+1)}\right]=2\cdot n$. | **1p** |
| | c) Conform punctului b), dând lui *n* valori naturale de la 1 la 43 se obţine:<br>$n=1\Rightarrow\left[\sqrt{1\cdot5}\right]=\left[\sqrt{1\cdot(4\cdot1+1)}\right]=2\cdot1$<br>$n=2\Rightarrow\left[\sqrt{2\cdot9}\right]=\left[\sqrt{2\cdot(4\cdot2+1)}\right]=2\cdot2$<br>$n=3\Rightarrow\left[\sqrt{3\cdot13}\right]=\left[\sqrt{3\cdot(4\cdot3+1)}\right]=2\cdot3$<br>.<br>.<br>.<br>$n=43\Rightarrow\left[\sqrt{43\cdot173}\right]=\left[\sqrt{43\cdot(4\cdot10+3)}\right]=2\cdot43$ | **2p** |
| | $\left[\sqrt{1\cdot5}\right]+\left[\sqrt{2\cdot9}\right]+\left[\sqrt{3\cdot13}\right]+...+\left[\sqrt{43\cdot173}\right]=2\cdot1+2\cdot2+2\cdot3+...+2\cdot43=$<br>$2\cdot(1+2+3+...+43)=43\cdot44=1892$. | **1p** |

| | | |
|---|---|---|
| 2. | a) Din inegalitatea mediilor $\Rightarrow a^4+1\ge 2a^2 \Rightarrow$ | **1p** |
| | $a^4+k^4+1\ge 2a^2+k^4\ge 2\cdot k^2\cdot\lvert a\rvert\cdot\sqrt{2}\ge 2\cdot k^2\cdot a\cdot\sqrt{2},\left(\forall a,k\in\mathbb{R}\right)$ | **2p** |
| | $b)\frac{1}{5+2^4}=\frac{1}{\left(\sqrt{2}\right)^4+2^4+1}\le\frac{1}{2\cdot\sqrt{2}\cdot 2^2\cdot\sqrt{2}}=\frac{1}{4\cdot 2^2}$ | |
| | $\frac{1}{5+3^4}=\frac{1}{\left(\sqrt{2}\right)^4+3^4+1}\le\frac{1}{2\cdot\sqrt{2}\cdot 3^2\cdot\sqrt{2}}=\frac{1}{4\cdot 3^2}$ | |
| | $\frac{1}{5+4^4}=\frac{1}{\left(\sqrt{2}\right)^4+4^4+1}\le\frac{1}{2\cdot\sqrt{2}\cdot 4^2\cdot\sqrt{2}}=\frac{1}{4\cdot 4^2}$ | **2p** |
| | . | |
| | . | |
| | . | |
| | $\frac{1}{5+n^4}=\frac{1}{\left(\sqrt{2}\right)^4+n^4+1}\le\frac{1}{2\cdot\sqrt{2}\cdot n^2\cdot\sqrt{2}}=\frac{1}{4\cdot n^2}$ | |
| | $\frac{1}{5+2^4}+\frac{1}{5+3^4}+\frac{1}{5+4^4}+\ldots+\frac{1}{5+n^4}\le\frac{1}{4\cdot 2^2}+\frac{1}{4\cdot 3^2}\frac{1}{4\cdot 4^2}+\ldots+\frac{1}{4\cdot n^2}=$ | |
| | $\frac{1}{4}\cdot\left(\frac{1}{2^2}+\frac{1}{3^2}+\ldots+\frac{1}{n^2}\right)<\frac{1}{4}\cdot\left(\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\ldots+\frac{1}{(n-1)\cdot n}\right)=$ | 1p |
| | $\frac{1}{4}\cdot\left(1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\ldots+\frac{1}{n-1}-\frac{1}{n}\right)=\frac{1}{4}\cdot\left(1-\frac{1}{n}\right)=$ | |
| | $\frac{1}{4}\cdot\frac{n-1}{n},\ (\forall)n\in\mathbb{N}^*-\{1\}$ | 1p |

**3.**

Fie MO $\cap$ BC=$\{P\}$, PB=PC; Fie Q $\in$ BF astfel încât BQ=AE=QF=3;

$$\left.\begin{array}{l} AE \perp (ABCD) \\ BF \perp (ABCD) \end{array}\right\} \Rightarrow AE \parallel BF;$$

$$\left.\begin{array}{l} AE = BQ \\ AE \parallel BQ \end{array}\right\} \Rightarrow ABQE \text{ dreptunghi} \Rightarrow EQ \parallel AB, EQ = AB = MP;$$ **1p**

$$\left.\begin{array}{l} EQ \parallel AB \\ EQ = MP \end{array}\right\} \Rightarrow EQPM \text{ paralelogram} \Rightarrow EM \parallel PQ,\ EM=PQ;$$ **1p**

$$\left.\begin{array}{l} EM=PQ \\ BP = PC \end{array}\right\} \Rightarrow PQ \text{ este linie mijlocie în } \triangle FBC \Rightarrow PQ \parallel CF$$ **1p**

$$\left.\begin{array}{l} MP \parallel DC \\ PQ \parallel CF \\ DC \cap CF = \{C\} \\ MP \cap PQ = \{P\} \end{array}\right\} \Rightarrow (EMPQ) \parallel (FCD).$$

*b*)

$$\left.\begin{array}{l} AB \parallel CD \\ CD \subset (CDF) \end{array}\right\} \Rightarrow AB \parallel (CDF) \Rightarrow d\big(A,(CDF)\big) = d\big(B,(CDF)\big);$$ **1p**

Fie BT $\perp FC, T \in FC$

$$\left.\begin{array}{l} BF \perp (ABC) \\ BC \perp CD \\ BC, CD \subset (ABC) \end{array}\right\} \overset{T.3.\perp}{\Rightarrow} FC \perp CD$$

$$\left.\begin{array}{l} BT \perp FC \\ TC \perp CD \\ BC \perp CD \\ FC, CD \subset (FCD) \end{array}\right\} \overset{R2.T.3.\perp}{\Rightarrow} BT \perp (FCD);$$ **1p**

$$\left.\begin{array}{l} \triangle FBC \\ m(\measuredangle(FBC)) = 90^\circ \\ BT \perp FC \\ BF = BC \end{array}\right\} \Rightarrow \left.\begin{array}{l} BT = \dfrac{FC}{2} \\ FC^2 = 6^2 + 6^2 \Rightarrow FC = 6\sqrt{2}cm \end{array}\right\} \Rightarrow BT = 3\sqrt{2}cm.$$ **1p**

*c*)

Fie PN $\perp FC$, N $\in$ FC

$$\left.\begin{array}{l} PN \perp FC \\ BT \perp FC \\ PN, BT \subset (FBC) \end{array}\right\} \Rightarrow PN \parallel BT,$$

$$\left.\begin{array}{l} PN \parallel BT \\ BT \perp (FBC) \end{array}\right\} \Rightarrow PN \perp (FBC) \Rightarrow PN = d\big((FBC),(EOM)\big);$$ 1p

$$\left.\begin{array}{l} PN \parallel BT \\ PB = PC \end{array}\right\} \Rightarrow PN \text{ linie mijlocie în } \triangle BTC \Rightarrow PN = \frac{BT}{2} = \frac{3\sqrt{2}}{2}cm$$

**4.**

Metoda 1.

Fie $BP \cap CD = \{M\}$; $DP \cap BC = \{N\}$

În $\triangle BCD$: $DN \cap CF \cap BM = \{P\} \overset{T.Ceva}{\Rightarrow} \frac{NB}{NC} \cdot \frac{CM}{MD} \cdot \frac{DF}{FB} = 1 \quad (1)$

$$\left.\begin{array}{l} DP \perp BM \\ [DN \text{ bisectoarea } \measuredangle BDC \end{array}\right\} \Rightarrow \triangle BDM \text{ isoscel} \Rightarrow BD = DM$$

$AC + BD = CD \Rightarrow AC = CD - BD \Rightarrow AC = CD - DM \Rightarrow AC = CM$

$\triangle BDC$: $\left[ DN \text{ bisectoarea } \Rightarrow \frac{NB}{NC} = \frac{BD}{DC} \right.$

Înlocuim în $(1) \Rightarrow \frac{BD}{DC} \cdot \frac{AC}{BD} \cdot \frac{DF}{FB} = 1 \Rightarrow \frac{DF}{FB} = \frac{DC}{AC}$

$\triangle ADC$: $\left[ CE \text{ bisectoarea } \measuredangle ACD \Rightarrow \frac{ED}{EA} = \frac{DC}{AC} \right.$

Rezultă că $\frac{DF}{FB} = \frac{ED}{EA} \overset{R.T.Th.}{\Rightarrow} EF \parallel AB$;

dar $AB \subset (ABC) \Rightarrow EF \parallel (ABC)$.

Metoda 2.

$\triangle DBM : DP \perp BM$, $[DP$ este bisectoarea $\measuredangle BDM \Rightarrow \triangle DBM$ este triunghi isoscel $\Rightarrow [BD] \equiv [DM]$, BP=PM ; **1p**

$$\left.\begin{array}{l} BD = DM \\ CD = AC + DB \end{array}\right\} \Rightarrow MC = AC \Rightarrow \triangle MAC \text{ este triunghi isoscel;}$$ **1p**

$$\left.\begin{array}{l} \triangle MAC \text{ este triunghi isoscel} \\ [CQ \text{ este bisectoarea } \measuredangle ACD,\ Q \in AM \end{array}\right\} \Rightarrow CQ \text{ este mediană} \Rightarrow$$

AQ=QM;

$$\left.\begin{array}{l} AQ = QM \\ BP = PM \end{array}\right\} \Rightarrow PQ \text{ este linie mijlocie în } \triangle AMB \Rightarrow PQ \parallel AB,$$ **1p**

$$\left.\begin{array}{l} PQ \parallel AB \\ AB \subset (ADB) \end{array}\right\} \Rightarrow PQ \parallel (ADB);$$ **1p**

$$\left.\begin{array}{l} PQ \parallel (ADB) \\ PQ \subset (CEF) \\ (CEF) \cap (ADB) = EF \end{array}\right\} \Rightarrow PQ \parallel EF;$$ **1p**

$$\left.\begin{array}{l} PQ \parallel EF \\ PQ \parallel AB \end{array}\right\} \Rightarrow EF \parallel AB$$ **1p**

$$\left.\begin{array}{l} EF \parallel AB \\ AB \subset (ABC) \end{array}\right\} \Rightarrow EF \parallel (ABC).$$ **1p**